6.4 Work – Quick Summary

	Metric	**Standard**
Mass	Kg	
Accel.	9.8 m/s^2	32 ft/s^2
Force	Newtons = N = kg·m/s^2	pounds = lbs
Dist.	m = meters	ft = feet
Work	Joules =J = N·m	foot-pounds = ft-lbs

Some unit facts

g = grams, kg = kilograms,
cm = centimeter
in = inches, yd = yards, mi = miles
1000 g = 1 kg
100 cm = 1 meter
12 inches = 1 foot
3 feet = 1 yard
5280 ft = 1 mile
force = mass · acceleration

Note: Given kilograms (mass), you must multiply by 9.8 m/s^2 to get corresponding the force (in Newtons) on Earth.
Pounds (lbs) is already a force, do NOT multiply by acceleration due to gravity.

Basic Work Concept: For a constant force moved a certain distance: $\text{Work} = \text{Force} \cdot \text{Distance}$
If force and/or distance are changing, then we find a pattern for force and distance and compute:

$$\text{Work} \;\; = \lim_{n\to\infty} \sum_{i=1}^{n} (FORCE)(DIST) = \int_a^b (FORCE)(DIST)$$

For all problems:
Step 0: Label and draw a picture of the start and end of the task.

***Problem type 1*: (“Leaky bucket”)** In these problems, the pattern for force is given or we can find it. The force changes every small moment (Dist = Δx) as the object is moved.

FORCE = f(x_i), DISTANCE = Δx, WORK = $\int_a^b f(x)dx$

Leaking at constant rate → $f(x) \; = \; mx + b$
Or force is given → $f(x)$ = force

Step 1: Find the formula for force.
Step 2: Integrate to get work.

***Problem type 2*: (“Stack of Books” - Chain/pumping)** In these problems, we find the weight of a slice at a given height and we find the formula for the distance that slice will move.

FORCE = weight of a horizontal slice = (density)(width of slice) or (density)(volume of slice)
DIST = distance moved by that slice

For chain:
- *k* = density = force per distance
- FORCE = weight of slice = kΔx
- DIST = distance moved by slice (typically *x* if you label like me)
- WORK = $\int_0^b x\ kdx$

For pumping:
- *k* = density = weight per volume
- FORCE = k volume = k(hor. slice area)Δy
- DIST = distance moved by slice (typically *a-y* if you label like me)
- WORK = $\int_0^b (a-y)k(horizontal\ slice\ area)dy$

Step 1: Label a typical horizontal slice. Find the formulas for weight of that slice and the distance that slice will move from start to finish.
Step 2: Integrate to get work.